

Model Question Paper -1

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute

Max. Marks : 100

Instructions :

The question paper has five parts namely A, B, C, D and E. Answer all the parts.

Use the graph sheet for the question on Linear programming in PART E.

PART - A

Answer ALL the questions

10 × 1=10

A relation R on A={1,2} defined by R={(1,1),(1,2),(2,1)} is not transitive, why?

Write the principal value branch of

Define a diagonal matrix.

If A is a square matrix of order 3 and $|A|=5$, then find $|adj(A)|$.

Differentiate the function $\tan^{-1} \sqrt{x}$ with respect to x.

Evaluate $\int \cos^2 x \sec^2 x dx$.

7. For what value of λ , is the vector $\lambda \mathbf{i} - \mathbf{j} + \mathbf{k}$ a unit vector?

8. Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$.

9. Define optimal solution in linear programming problem.

10. If $P(A) = 0$ and $P(B) = \frac{1}{2}$, then find $P(A \cap B)$ if exists.

PART B

Answer any TEN questions:

10 × 2=20

11. Find the gof and fog if $f(x) = \frac{1}{x}$ and $g(x) = x^2$.

12. Write the function $\frac{1}{x^2+1}$, in the simplest form.

13. Prove that $\sin^2 x + \cos^2 x = 1$.

14. If area of the triangle with vertices $(-2, 0)$, $(0, 4)$ and $(0, k)$ is 4 square units, find the value of „k“ using determinants.

15. Find $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$, if $\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = \frac{\pi}{4}$, where $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$.

16. If $x = 2at$, $y = -2at^2$. Find $\frac{dy}{dx}$.

17. Show that the function $f(x) = \frac{1}{x}$ given by $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$ is strictly increasing on $(-\infty, 0)$.

18. $\int \frac{dx}{\sin^2 x \cos^2 x}$

Find $\int e^x \sec x \sqrt{1 + \tan x} dx$.

19. Evaluate: $\int \frac{dx}{\sin^2 x \cos^2 x}$.

20. Find the order and degree of the differential equation

$y'' = x$

21. The position vectors of two points P and Q are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively. Find the position vector of a point R which divides the line segment PQ in the ratio 2 : 1 internally.

22. Prove that $[a, b, c + d] = [a, b, c] + [a, b, d]$

23. Find the angle between the pair of lines $r = 3\hat{i} + 5\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$ and $r = 7\hat{i} + 4\hat{k} + \mu(2\hat{i} + 2\hat{j} + 2\hat{k})$.

24. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, Find $P(A \cup B)$.

PART C

Answer any TEN questions:

10 × 3 = 30

25. If a and b are defined as $a \cdot b = |a \times b|$ and $a \cdot (b \cdot c) = a \cdot b \cdot c$, Show that \cdot is commutative but not associative and \times is associative.

Prove that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Express O_1 as the sum of a symmetric and skew symmetric matrices.

28. If $y = \frac{1}{x}$, find $\frac{dy}{dx}$

29. If $y = \frac{1}{x}$, find $\frac{d^2y}{dx^2}$

30. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

31. $\int \frac{dx}{x^2 + 1}$

32. Evaluate $\int_0^1 x dx$ as the limit of sum.

33. Find the area between the curves $y = x$ and $y = x^2$.

34. For the differential equation $xy \frac{dy}{dx} = (x + 2)(y + 2)$, find the solution curve passing through the point (1, -1).

35. Find the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$,

where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

36. If \vec{a} and \vec{b} are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

37. Find the angle between the line _____ and the plane _____

38. Two dice are thrown simultaneously. If X denotes the number of sixes. Find the mean (expectation) of X .

PART D

Answer any SIX questions:

6 × 5=30

39. If _____ is the set of all non-negative real numbers prove that the function _____ is invertible. Write also _____

40. If $A = [\dots]$, $B = [\dots]$ - Verify that $(AB)^T = B^T A^T$

Solve the following system of equation by using matrix method:

$$x + y + z = 6, y + 3z - 11 = 0 \text{ and } x + z = 2y.$$

42. If _____

The volume of a cube is increasing at a rate of 9cc/sec. How fast is the surface area increasing when the length of an edge is 10 cm.

44. Find the integral of _____ with respect to _____ and hence _____

45. Find the area bounded by the curve _____ and the line _____

Solve the differential equation, _____

Derive the equation of a line in space passing through two given points both in the vector and Cartesian form.

If a fair coin is tossed 6 times. Find the probability of (i) at least five heads and (ii) at most five heads (iii) exactly 5 heads.

PART E

Answer any ONE question:

1 × 10=10

49. (a) Prove that $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ and evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

(b) Prove that
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

50. (a) Solve the following linear programming problem graphically: Minimize and maximize _____, subject to constraints _____

(b) Discuss the continuity of the function _____

MODEL QUESTION PAPER – 2

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute

Max. Marks : 100

Instructions :

The question paper has five parts namely A, B, C, D and E. Answer all the parts.
Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions

10 × 1=10

1. Define a binary operation on a set.
2. Write the range of $f(x)=\sin^{-1}x$ in $[0,2\pi]$ other than 0.
3. If a matrix has 7 elements, write all possible orders it can have.
4. If A is a square matrix of order 3 and $|A|=4$, then find $|3A|$.
5. If $y=e^{\log x}$, Show that $\frac{dy}{dx} = 1$.
6. Evaluate $\int \frac{1}{x^2} dx$.
7. If \hat{i} is a unit vector such that $(\hat{i} \cdot \hat{j}) = (\hat{j} \cdot \hat{k})$, find $|\hat{j}|$.

Find the equation of plane with the intercepts 2, 3 and 4 on x, y and z axis respectively

Define Optimal Solution in Linear Programming Problem.

A fair die is rolled. Consider the events $E=\{1,3,5\}$ and $F=\{2,3\}$, find $P(E|F)$.

PART B

Answer any TEN questions:

10 × 2=20

11. Define an equivalence relation and give an example.
12. Prove that $3 \cos^2 x + \sin^2 x = 2 + 2 \cos^2 x$.
13. Write in the simplest form of $\tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, $0 < x < \pi$.
If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1} .

15. Prove that the function f given by $f(x) = |x - 1|$, $x \in R$ is not differentiable at $x = 1$.

Find „c“ of the mean value theorem for the function $f(x) = 2x^2 - 10x + 29$ in $[2, 7]$.

17. Find a point on the curve $y = \sqrt{x}$ at which the tangent is parallel to the x-axis.

18. Evaluate $f'(x)$ at $x = 1$.

19. Find $f'(x)$ at $x = 2$.

20. Form the differential equation of the family of curve $y = a(x^2 + 1)$.

21. Find the unit vector in the direction of $a = i - 2j$, also find the vector whose magnitude is 7 units and in the direction a .

22. If $\vec{u} = \sqrt{2}i + \sqrt{2}j$ and $\vec{v} = \sqrt{2}i - \sqrt{2}j$, find the angle between \vec{u} and \vec{v} .

Find the angle between the pair of planes $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$.

Find the probability distribution of number of heads in two tosses of a coin.

PART C

Answer any TEN questions:

10 × 3 = 30

25. If $f: N \rightarrow N$, $g: N \rightarrow N$ and $h: N \rightarrow N$ defined as $f(x) = 2x$, $g(y) = 3y + 4$ and $h(x) = x^2$, x, y, z in N . Show that $h \circ (g \circ f) = (h \circ f) \circ g$.

Show that $\sin^{-1} \frac{1}{3} + \cos^{-1} \frac{2}{5} + \tan^{-1} \frac{1}{6} = \frac{\pi}{2}$.

27. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, show that $A^2 = \begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$ and $AA^T = I$.

Find $\frac{dy}{dx}$, if $x^3 + x^2y + xy^2 + y^3 = 81$.

29. Differentiate $(\sin x)^2$ with respect to x .

Find the absolute maximum value and the absolute minimum value of the function $f(x) = \sin x + \cos x$, $x \in [0, \pi]$.

31. Find $\int \frac{xe^x}{(1+x)^2} dx$ _____
 Evaluate: $f'(0)$

Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

34. Find the general solution of $\frac{dy}{dx} + y = 1$ ($y = \dots$).

Find the value of p so that the lines \dots are at right angles.

Find the area of the rectangle having vertices A, B, C and D with P, V respectively.

Show that the four points A, B, C and D with position vectors $4i+5j+k$, $(j+k)$, $3i+9j+4k$ and $-4i+4j+4k$ respectively coplanar.

In answering a question on a multiple choice test a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{4}$. What is the probability that a student knows the answer given that he answered it correctly.

PART D

Answer any SIX questions:

6 × 5 = 30

3, show that f is

39. If $f(x)$ defined by $f(x) = \dots$, where A is invertible and $B = \dots$.

40. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$ Prove that $(AB)C = A(BC)$

41. Solve by matrix method:

If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y'' + xy' + y = 0$.

43. A ladder $10m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of $2m/s$.

How fast is its height on the wall decreasing when the foot of the ladder is $6m$ from the wall?

is away from the wall?
 44. Find the integral of \sqrt{f} with respect to x and hence evaluate $\int \sqrt{f} dx$

Find the area of the smaller region enclosed by the circle $x^2+y^2=4$ and the line $x+y=2$ by integration method.

46. Solve the differential equation $y'' = \sin x$, $y=0$ when $x=0$

Derive the equation of a plane perpendicular to a given vector and passing through a given point in both vector and Cartesian form.

In an examination 20 questions of true-false are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, the answer is "true", if it falls tails, the answer is "false". Find the probabilities that he answers at least 12 questions correctly.

PART E

Answer any ONE question:

1 × 10 = 10

(a) One kind of cake requires 200 gm of flour and 25 g of fat and another kind of cake requires 100 gm of flour and 50 gm of fat. Find the maximum number of cakes which can be made from 5 kg and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

(b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

50. (a) Prove that $\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$

and hence evaluate $\int_{-1}^1 x^2 dx$

Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

MODEL QUESTION PAPER – 3

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute

Max. Marks : 100

Instructions :

The question paper has five parts namely A, B, C, D and E. Answer all the parts.
Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer ALL the questions

10 × 1=10

Let * be a binary operation on N given by $a*b = \text{LCM of } a \text{ and } b$. Find $20*16$.

What is the reflection of the graph of the function $y = \sin x$ along the line $y = x$.

What is the number of possible square matrices of order 3 with each entry 0 or 1?

4. For what value of x, the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ are singular.

Write the derivative of $\sin^{-1}(\cos x)$ with respect to x.
Evaluate $\int dx$

7. Find if the vector \vec{a} and \vec{b} are perpendicular to each other.

Write the vector form of the equation of the line

Define optimal solution in Linear programming problem.

If $P(A) = 0.3$, $P(\text{not } B) = 0.4$ and A and B are independent events, find $P(A \text{ and not } B)$.

PART B

Answer any TEN questions

10 × 2=20

11. Show that the signum function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

12. Find the value of k, if $A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A_2 = kA_1$, then write the value of k.

14. If $x = \sqrt{a^2 - y^2}$, then Show that $\frac{dx}{dy} = -\frac{y}{x}$

15. Write the inverse trigonometric function $\sin^{-1} \left(\frac{x}{a} \right)$, $|x| < a$, in simplest form.

If $\sin 2x + \cos 2y = 1$, Show that $x + y = \frac{\pi}{2}$

If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area.

18. Evaluate $f(x) = \int_0^x (x-t)^2 dt$

19. Find $f(x) = \int_0^x \sqrt{t} dt$

20. Verify that $y = \sqrt{x}$ is a solution of the differential equation $xy'' + y' = 0$

21. Find the magnitude of two vector \vec{a} and \vec{b} and their scalar product is $\vec{a} \cdot \vec{b} = 1$

22. Show that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$

23. If the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-1}{4} = \frac{z-1}{-5}$ are perpendicular, find the value of k.

A die is tossed thrice. Find the probability of getting an odd number at least once.

PART C

Answer any TEN questions

10 × 3=30

25. Show that the relation R in the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} + is given by $R = \{(a,b) : |a-b| \text{ is a multiple of } 4\}$ is an equivalence relation.

26. Prove that $\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$

27. Using elementary operations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

28. If $y = (\log x)^{\cos x} + x^{\sin x}$ find $\frac{dy}{dx}$

29. If $x = \cos^{-1} \left(\frac{1}{2} \right)$ and $y = \sin^{-1} \left(\frac{1}{2} \right)$, Show that $x + y = \frac{\pi}{2}$.

30. At what points, the function $f(x) = \sin x - \cos x$, $x \in [0, 2\pi]$, attains local maxima and minima

31. Evaluate $\int_0^1 x \cos x dx$

32. Find $\int \frac{1}{x^2 + 1} dx$

Find the area of the region enclosed by the circle $x^2+y^2=a^2$ by integration method.

Solve the differential equation — — —

35. Show that the points $(-1, 1), (1, 1), (1, -1)$ are the vertices of right angled triangle.

36. Three vectors satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ satisfy the condition

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$, Find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$.

A random variable x has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine (i) k (ii) $p(x < 3)$

PART D

Answer any SIX questions

6 × 5=30

Let R_+ be the set of all non negative real numbers, Show that the function $f:R_+ \rightarrow [4,\infty)$ defined by $f(x)=x^2 + 4$ is invertible. Also find the inverse of f(x)

40. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ verify that A is a skew symmetric matrix and A^2 is a symmetric matrix.

41. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of equations

$$2x - 3y + 5z = 11; 3x + 2y - 4z = -5 \text{ and } x + y - 2z = -3$$

42. If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

43. Sand is pouring from a pipe at the rate of . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is ?

44. Find the integral of _____ with respect to x and evaluate \int_0^1 _____ .

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Derive an equation of a plane in the normal form both in vector and Cartesian form.

47. Solve $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$ given that $y=0$ when $x=0$

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs i) none ii) not more than one iii) more than one iv) at least once will fuse after 150 days of use?

PART E

Answer any ONE question

1 × 10 = 10

49. (a) Prove that $\int_0^1 (x^2 + 1) dx = \frac{4}{3}$ and hence evaluate

$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$

(b) Show that

(a) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

(b) Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

$\begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$

